Foundations of Adaptor Signatures

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Once Upon A Time

Alice
(wants to buy a witness for a statement Y)

$$Y$$

Bob
(knows a witness for Y and wants to sell it)

$$y$$

wants to rely on minimum trust

does not like the ROM

Adaptor Sigantures

Adaptor Signature Interfaces

Signature Scheme

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KGen}(\lambda)$

$\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, m)$

$b \leftarrow \mathsf{Vrfy}(\mathsf{pk}, m, \sigma)$

Hard Relation

$(Y, y) \leftarrow \mathsf{GenR}(\lambda)$

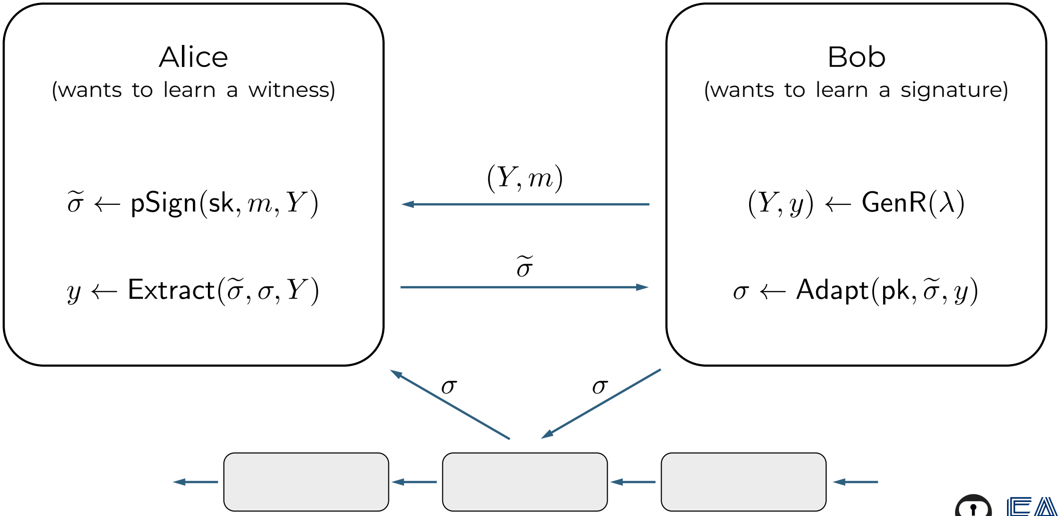
$\widetilde{\sigma} \leftarrow \mathsf{pSign}(\mathsf{sk}, m, Y)$

$b \leftarrow \mathsf{pVrfy}(\mathsf{pk}, m, \widetilde{\sigma}, Y)$

$\sigma \leftarrow \mathsf{Adapt}(\mathsf{pk}, \widetilde{\sigma}, y)$

$y \leftarrow \mathsf{Extract}(\widetilde{\sigma}, \sigma, Y)$

Fair Exchange using Adaptor Signatures



Adaptor Signatures in the Literature

- Introduced by Andrew Poelstra 2017
- Formally defined by Aumayr et al. [AEEFHMMR'21]
- Applications:
  - (Generalized) Payment Channels [AEEFHMMR'21]
  - (Blind) Coin Mixing [GMMMTT'22, QPMSESELYY'23]
  - Oracle-Based Payments [MTVFMM'23]
- Theory:
  - PQ Adaptors [TMM'20]
  - Stronger Definitions [DOY'22]

Theoretical Challenges

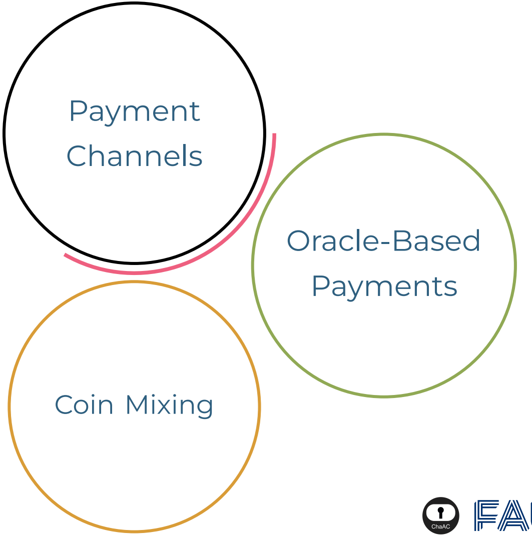
Given a signature scheme, building a secure adaptor signature is hard.

There is no secure adaptor signature in the standard model.

Practical Challenges

Adaptor signatures were formalized to build **payment channels**.

This formalization **does not match** the most recent applications.



Our Contribution

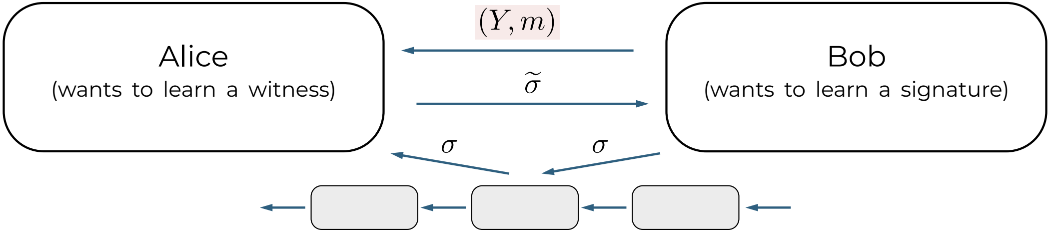
Gaps

Definitions

Constructions

Transparent Reductions

Adaptor Signature Formalization



- The definition is a **one-shot experiment**
  - The adversary can only learn a single challenge pre-signature
- Adaptor signatures achieve only **existential unforgeability**, even if the signature scheme is strongly unforgeable
- The pre-signer **cannot influence** the statement

Oracle-Based Conditional Payments [MTVFMS'22]

Alice

sends a payment when the oracle testifies for an event

$\forall i \in \{1, \ldots, M\}:$

$(Y_i, y_i) \leftarrow \mathsf{RGen}(1^\lambda)$

$\forall i \in \{1, \ldots, M\}:$

$\widetilde{\sigma}_i \leftarrow \mathsf{pSign}(\mathsf{sk}, m, Y_i)$

$(y_1, \ldots, y_N)$ →

Oracles

testify for events

~~y_i~~

$\widetilde{\sigma}_{1 \leq i \leq M}$ →

Bob

obtains pre-signatures from Alice and requests the oracle for testimony

~~$\sigma \leftarrow \mathsf{Adapt}(\mathsf{pk}, \widetilde{\sigma}_i, y_i)$~~

$\sigma \leftarrow \widetilde{\sigma}_1 \oplus \widetilde{\sigma}_2$

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Theoretical Challenges

Can we generically transform signatures into adaptor signatures?

Can we find an adaptor signature scheme in the standard model?

Dichotomic Signatures: Pre-Signing

$\mathsf{pSign}(\mathsf{sk}, m, Y)$

$1: r \leftarrow\!\!\$\ \mathbb{Z}_p; R \leftarrow g^r$

$2: h \leftarrow \mathcal{H}(\mathsf{pk}, R \cdot Y, m)$

$3: \mathbf{return}\ (R \cdot Y, \mathsf{sk} \cdot h + r)$

- The signature consists of two parts

$$\sigma = (\sigma_1, \sigma_2)$$

- The signature uses a homomorphic one-way function

$$R = \mathsf{OWF}(r); Y = \mathsf{OWF}(y); r, y \in \mathbb{Z}_p$$

- One part can be computed using

$$\sigma_1 = \Sigma_1(\mathsf{sk}, m; \mathsf{OWF}(r) \cdot \mathsf{OWF}(y))$$

- The other part can be computed using

$$\sigma_2 = \Sigma_2(\mathsf{sk}, m; r)$$

Dichotomic Signatures: Adapt/Extract

$\mathsf{Adapt}(\mathsf{pk}, \widetilde{\sigma}, y)$

1 : parse $\widetilde{\sigma}$ as $(\widetilde{\sigma}_1, \widetilde{\sigma}_2)$

2 : **return** $(\widetilde{\sigma}_1, \widetilde{\sigma}_2 + y)$

$\mathsf{Extract}(Y, \widetilde{\sigma}, \sigma)$

1 : parse $\widetilde{\sigma}$ as $(\widetilde{\sigma}_1, \widetilde{\sigma}_2)$

2 : parse σ as (σ_1, σ_2)

3 : **return** $\sigma_2 - \widetilde{\sigma}_2$

- The second part of the signature is homomorphic in the randomness

$$\Sigma_2(\mathsf{sk}, m; r) + y = \Sigma_2(\mathsf{sk}, m; r + y)$$

Dichotomic Signatures: A Definition

A signature scheme w.r.t. a homomorphic one-way function OWF is dichotomic; if

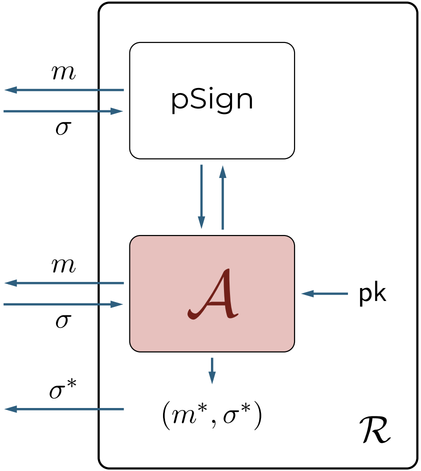
- It is decomposable

$$\sigma = (\sigma_1, \sigma_2) = (\Sigma_1(\mathsf{sk}, m; \mathsf{OWF}(r)), \Sigma_2(\mathsf{sk}, m; r))$$

- It is homomorphic in the randomness

$$\Sigma_2(\mathsf{sk}, m; r) + y = \Sigma_2(\mathsf{sk}, m; r + y)$$

Proving Security



- We need to simulate pre-signatures to the adversary
- We cannot use the random oracle

> Converting a signature into a pre-signature seems impossible

- We **cannot** reduce to the strong unforgeability directly

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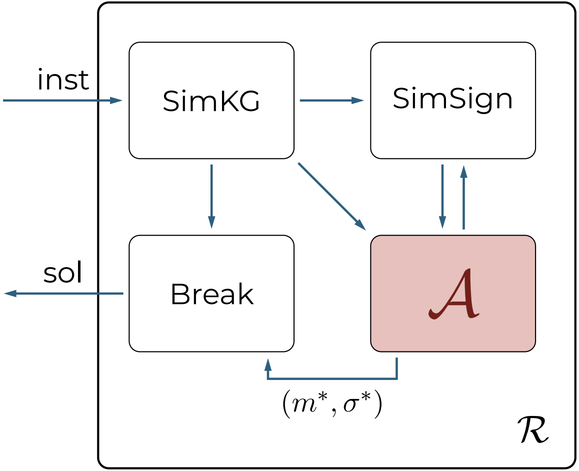
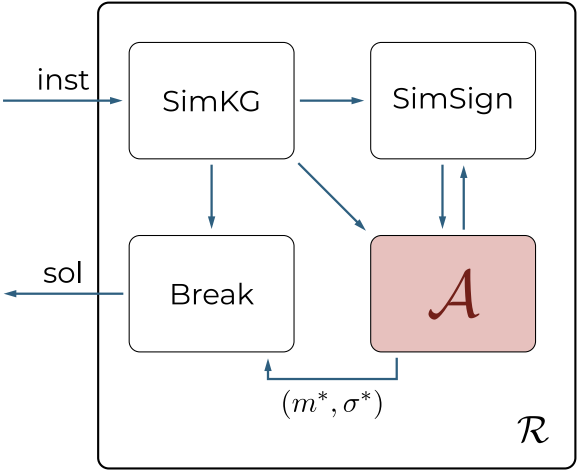
Transparent Reductions



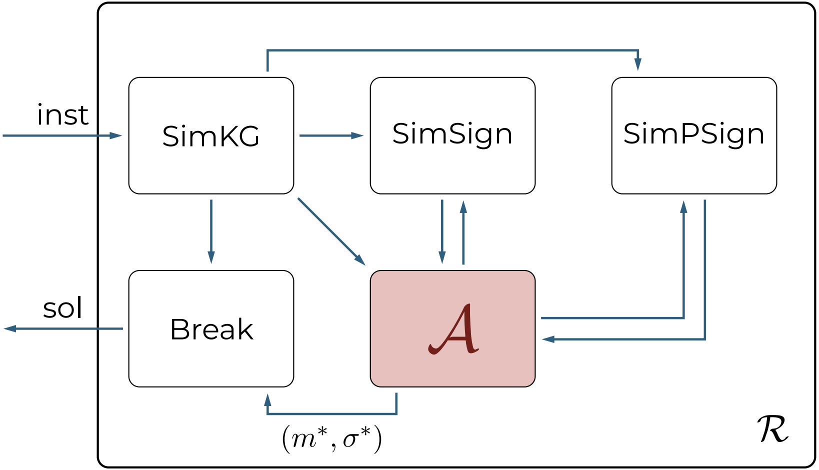
- **SimKG**:
  - Simulates keys $(\mathsf{simSK}, \mathsf{simPK})$
- **SimSign**:
  - Simulates signatures using simSK
- **Break**:
  - Solve problem instance using valid forgery

Simulating Pre Signatures



- So far, we **can**:
  - Simulate keys
  - Provide a signature oracle
  - Break the problem instance using a forgery
- So far, we **cannot**:
  - Provide a pre-signature oracle

Simulatable Transparent Reductions



A Framework For Adaptor Signatures

A secure adaptor signature scheme requires the following three checks:

- The signature scheme is **dichotomic**
- There is a **transparent reduction** from the strong unforgeability to an underlying hard problem
- We can simulate a pre-signature oracle **(simulatability)**

Conclusion

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